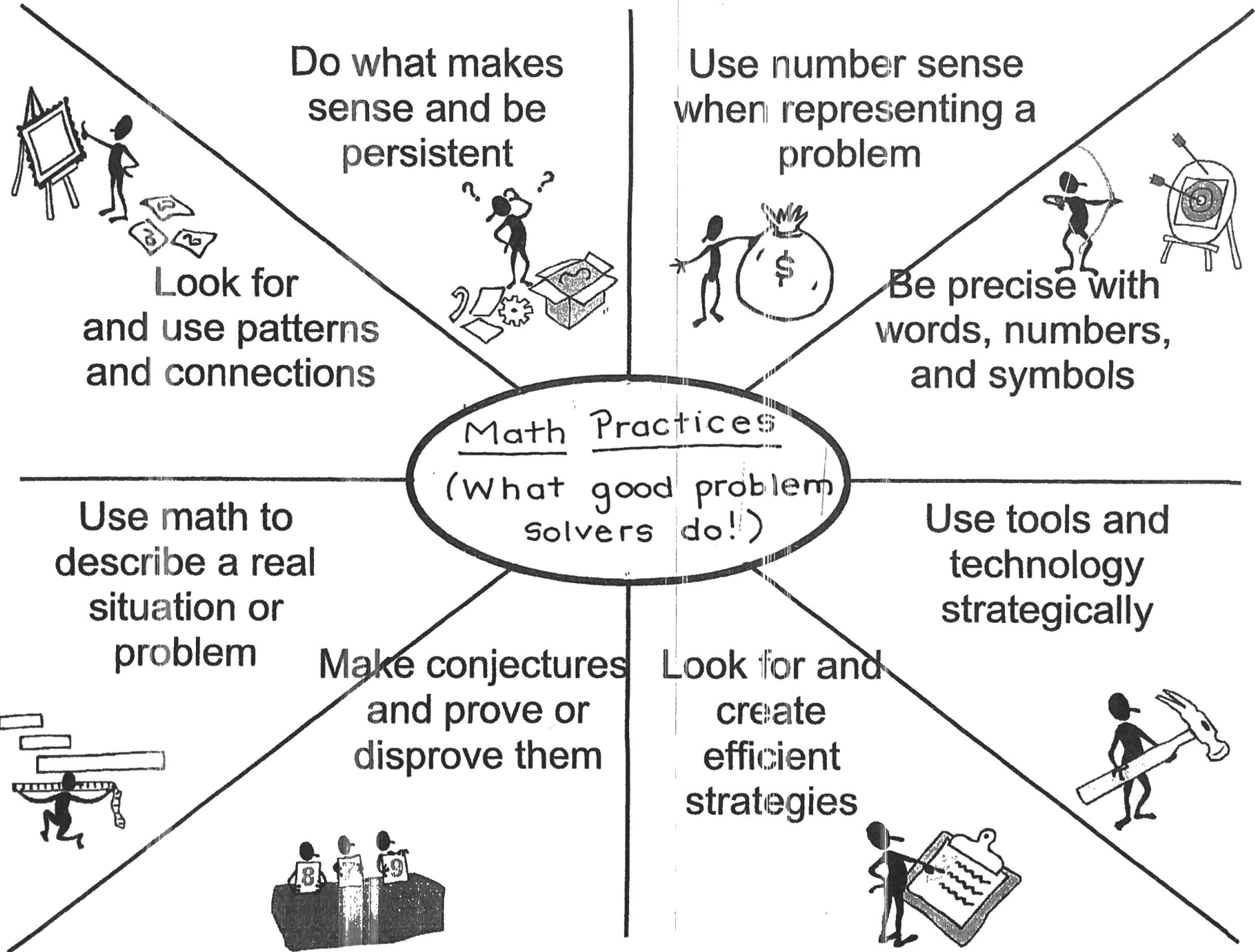


COMMON CORE STANDARDS FOR MATHEMATICAL PRACTICES

The Common Core State Standards for Mathematical Practice are expected to be integrated into every mathematics lesson for all students Grades K-12.

Mathematics Practice Standards Explanations and Examples:

- 1. Make sense of problems and persevere in solving them.**
- 2. Reason abstractly and quantitatively.**
- 3. Construct viable arguments and critique the reasoning of others.**
- 4. Model with mathematics**
 - Mathematically proficient students in grade 4 experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, making a chart, list, or graph, creating equations, etc. Students need opportunities to connect the different representations and explain the connections. They should be able to use all of these representations as needed. Fourth graders should evaluate their results in the context of the situation and reflect on whether the results make sense.
 - Mathematically proficient students in grade 5 experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, making a chart, list, or graph, creating equations, etc. Students need opportunities to connect the different representations and explain the connections. They should be able to use all of these representations as needed. Fifth graders should evaluate their results in the context of the situation and whether the results make sense. They also evaluate the utility of models to determine which models are most useful and efficient to solve problems.
- 5. Use appropriate tools strategically.**
- 6. Attend to precision.**
- 7. Look for and make use of structure.**
- 8. Look for and express regularity in repeated reasoning.**

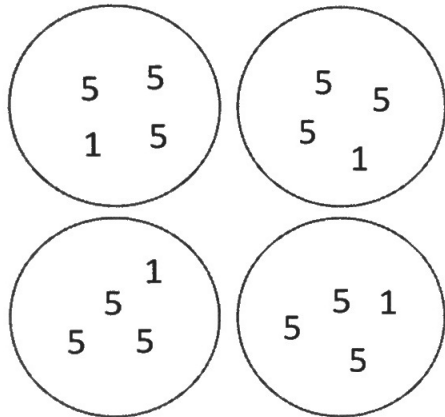


Grades 4 & 5 Progression of Multiplication Strategies

Area Model	Partial-Products	U.S. Standard Algorithm												
34×28 <table style="margin: auto; border-collapse: collapse;"> <tr> <td style="padding: 5px;"></td> <td style="padding: 5px; text-align: center;">20</td> <td style="padding: 5px; text-align: center;">8</td> <td style="padding: 5px;"></td> </tr> <tr> <td style="padding: 5px; text-align: right;">30</td> <td style="border: 1px solid black; padding: 5px; text-align: center;">600</td> <td style="border: 1px solid black; padding: 5px; text-align: center;">240</td> <td style="padding: 5px;"></td> </tr> <tr> <td style="padding: 5px; text-align: right;">4</td> <td style="border: 1px solid black; padding: 5px; text-align: center;">80</td> <td style="border: 1px solid black; padding: 5px; text-align: center;">32</td> <td style="padding: 5px;"></td> </tr> </table> <div style="margin-left: 150px; text-align: right;"> $\begin{array}{r} 32 \\ 80 \\ 240 \\ + 600 \\ \hline 952 \end{array}$ </div>		20	8		30	600	240		4	80	32		34×28 $\begin{array}{r} 28 \\ \times 34 \\ \hline 32 \quad (4 \times 8) \\ 80 \quad (4 \times 20) \\ 240 \quad (30 \times 8) \\ + 600 \quad (30 \times 20) \\ \hline 952 \end{array}$	34×28 $\begin{array}{r} 3 \\ 28 \\ \times 34 \\ \hline 112 \\ 840 \\ + \\ \hline 952 \end{array}$
	20	8												
30	600	240												
4	80	32												
<p>This strategy provides a good visual model for multiplication, relating it to the process for finding the area of a rectangle. The visual model helps ensure that students remember to multiply every digit of one factor by every digit of the other factor (using the distributive property).</p>	<p>Like the <i>Area Model</i>, this strategy uses the distributive property. Students multiply every digit of one factor by every digit of the other factor; then add the “partial-products” together. This is a more abstract representation of the exact same steps shown in the <i>Area Model</i> and a more concrete representation of the exact steps shown in the <i>U.S. Standard Algorithm</i>.</p> <p>Students should be able to do this by the end of grade 4.</p>	<p>This strategy is required in fifth grade by standard 5.NBT.5. It follows the exact same steps as “Partial-Products,” but uses a more compact notation to make it more efficient. This efficiency, however, also obscures the place value of the numbers and the logic of the steps. Students will come to understand the logic of each step and the meaning of the shorthand much better when they follow the progression of strategies depicted here. Therefore, this algorithm should not be introduced prematurely.</p> <p>Students <u>must</u> be able to do this by the end of grade 5.</p>												

Grades 3-5 Progression of Division Strategies

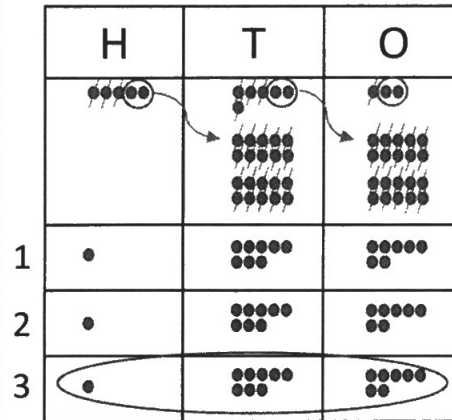
**3rd grade
Equal Groups**



$64 \div 4 = 16$

As students begin to build their understanding of what division means, this model helps students recognize the relationships between the number of groups, the size of each group, and the total. Work in this model might begin with students "handing out" the total into a given number of equal groups (partitive contexts). Alternatively, students might be given the size of the group, and use this model to identify the number of groups (quotative contexts). Later, students develop flexibility in how they are distributing into groups, thinking in larger groups of tens and ones rather than by just ones.

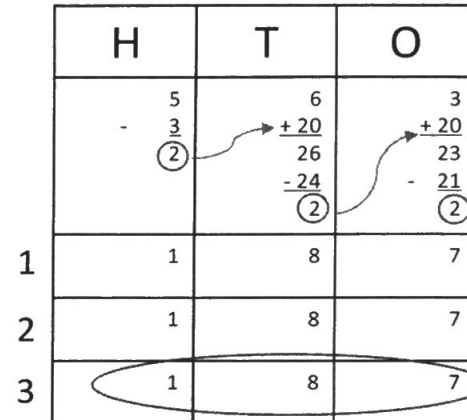
**4th and 5th grade
Place Value Discs/Dots**



$563 \div 3 = 187 \text{ r}2$

Place value discs refer to coin shaped manipulatives that are printed with the place value to which they belong on them. Students use the physical discs and an equal grouping structure to model distributing the values equally across a place value chart. Students use their understanding that each place value moving left is ten times the value of the current place to develop the need for "unbundling" when there are units remaining in a place value. Place value dots work in the same way, and are a written representation of the place value discs.

**4th and 5th grade
Place Value Digits**



$563 \div 3 = 187 \text{ r}2$

Place value digits are an abstracted form of the discs and dots. Students are now relying on their place value understanding to use digits in the chart to represent the amount in each place value. The "unbundling" is represented as the converting of leftovers in one place value to ten times that number in the place to the right. Here, students will begin to see parts of their thinking closely resemble the work of the standard algorithm. In fifth grade, students will begin again with this model when they unbundle remaining ones into decimal place values.

**5th grade
Standard Algorithm**

$$\begin{array}{r}
 140 \\
 4 \overline{) 562} \\
 \underline{-4} \\
 16 \\
 \underline{-16} \\
 02 \\
 \underline{-0} \\
 2
 \end{array}$$

The standard algorithm, now grounded in many experiences with place value-based representations, is our end goal for representing division. It is the most efficient strategy, and students can explain how it connects to the previous representations. At this stage, students understand that the "bring down" step works as a result of unbundling from a larger place value to the next smaller and combining the values. Students in fifth grade use their powers of ten understanding to work flexibly with decimal forms in both the divisor and the dividend.